Essence of Basic Transformations

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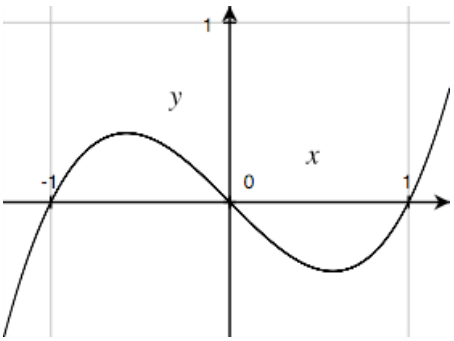
Note: *v* represents one variable. In two dimensions, *v* could be *x* or *y*.

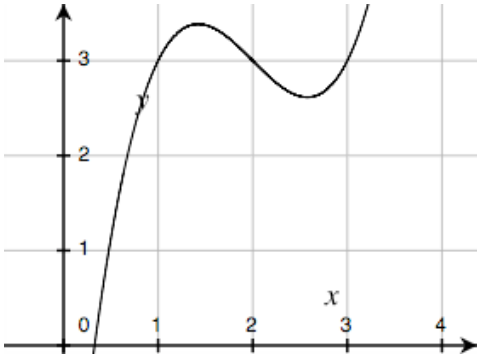
To	**Replace *v*-values in all points by**	**Replace all parent relation's *v*-variables by**
translate c units in v direction	$v + c$	$v - c$
dilate by c in v direction	cv	v/c; $0 < c < 1$ shrinks, $1 < c$ stretches*
reflect in $v = 0$	$-v$	$-v$
reflect in origin	reflect in $v = 0$ for all v	
reflect in $y = x$	$(x, y) \to (y, x)$ in relation and all points. (Do this to get inverses. E.g., $y = e^x \to x = e^y \Leftrightarrow y = \ln x$.)	
rotate +90°	$(x, y) \to (-y, x)$ in relation and all points. (If m is slope, then for lines: $l_1 \perp l_2 \Leftrightarrow m_2 = -1/m_1$)	

Examples:

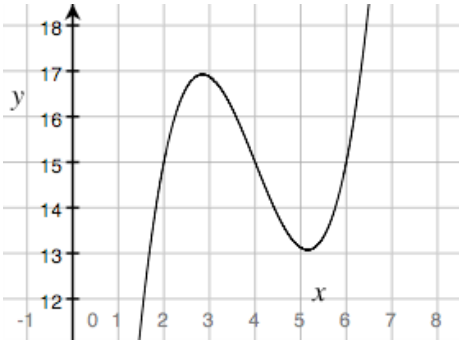
1. Parent, $y = x(x+1)(x-1)$:



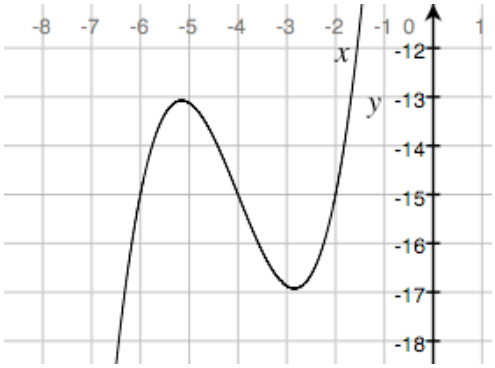
2. Translate up 3, right 2 (translation vector: <2,3>), $y - 3 = (x-2)(x-2+1)(x-2-1)$:



3. Stretch vertically 5 units and horizontally 2, $y/5 - 3 = (x/2-2)(x/2-2+1)(x/2-2-1)$



4. Reflect in origin (note axes scales): $-y/5 - 3 = (-x/2-2)(-x/2-2+1)(-x/2-2-1)$



* Some texts treat stretching in *x* and *y* directions differently. In Blitzer, Precalculus, 2nd Ed, $y = f(x) \to y = c_y f(c_x x)$, where in the x direction $0 < c_x < 1$ stretches and $1 < c_x$ shrinks.