Addition Table

10	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
9	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
8	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
7	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
6	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
1	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
0	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
-1	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
-2	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
-3	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
-4	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-5	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
-6	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
-7	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
-8	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-9	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
-10	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
+	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

The Essentials

Identity Property of Addition: $a + 0 = a$. So zero (0) is called the Additive Identity Element.
Inverse Property of Addition: $a + (-a) = 0$. a and $-a$ are called opposites of each other.
Subtraction is the addition of the opposite: $a - b = a + (-b) = d$. Check: $a = b + d$.

148 + 359 + 67	= 100 + 40 + 8 + 300 + 50 + 9 + 60 + 7	10^*n* place value
	= 100 + 300 + 40 + 50 + 60 + 8 + 9 + 7	commutative prop. (+)
12 <-- *carries*	= (1 + 3) x 100 + (4 + 5 + 6) x 10 + (8 + 9 + 7) x 1	distributative prop.
148	= (1 + 3) x 100 + (4 + 5 + 6) x 10 + (17 + 7) x 1	addition
359	= (1 + 3) x 100 + (4 + 5 + 6) x 10 + (10 + 7 + 7) x 1	10^*n* place value
67	= (1 + 3) x 100 + (4 + 5 + 6) x 10 + (1) x 10 + (7 + 7) x 1	distributative prop.
574	= (1 + 3) x 100 + (4 + 5 + 6) x 10 + (1) x 10 + (14) x 1	addition
	= (1 + 3) x 100 + (4 + 5 + 6) x 10 + (1 + 1) x 10 + (4) x 1	distributative prop.
4T <-- borrow	= (1 + 3) x 100 + (4 + 5 + 6 + 2) x 10 + (4) x 1	carry
6T <-- borrow	= (1 + 3) x 100 + (17) x 10 + (4) x 1	addition
~~5~~74	= (1 + 3) x 100 + (10 + 7) x 10 + (4) x 1	10^*n* place value
(–) 289	= (1 + 3) x 100 + (1) x 100 + (7) x 10 + (4) x 1	distributative prop.
285	= (1 + 3 + 1) x 100 + (7) x 10 + (4) x 1	carry
	= (5) x 100 + (7) x 10 + (4) x 1	addition
T := 10	= 574	10^*n* place value

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Multiplication Table

10	-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100
9	-90	-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90
8	-80	-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80
7	-70	-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70
6	-60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60
5	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50
4	-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40
3	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30
2	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20
1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
-2	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20
-3	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30
-4	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40
-5	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
-6	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54	-60
-7	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63	-70
-8	80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72	-80
-9	90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90
-10	100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100
X	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

The Essentials

Identity Property of Multiplication: $n \times 1 = n$. So one (1) is called the Multiplicative Identity Element.
Inverse Property of Multiplication: $n \times (1/n) = 1$. $1/n$ is called the Multiplicative Inverse of n.
Division is multiplication by the inverse: $n / d = n \times (1/d) = q + r / d$. Check: $n = dq + r$.

358920.25 / 372.1 = 3589202.5 / 3721

```
                964.58  <-- (+)
3721 3589202.50 = remaining to divide
  (–) 3348900.00 = 3721 x 900
      240302.50 = remaining to divide
   (-) 223260.00 = 3721 x 60
       17042.50 = remaining to divide
    (-) 14884.00 = 3721 x 4
        2158.50 = remaining to divide
     (-) 1860.50 = 3721 x 0.5
         298.00 = remaining to divide
      (-) 297.68 = 3721 x 0.08
           0.32 = remaining to divide
```

Each multiplier is chosen by trial and error to produce the largest product less than or equal to the remaining to divide.

358920.25 =? 372.1 x 964.58 + 0.032

```
   111 2   <-- x 300 carries
   434 5   <-- x 70 carries
   1  1 1   <-- x 2 carries
    964.58
  (x) 372.1  <-- (+)
    96.458 = 964.58 x 0.1
(+) 1929.160 = 964.58 x 2
(+) 67520.600 = 964.58 x 70
(+) 289374.000 = 964.58 x 300
  358920.218
   (+)  0.032
  358920.250 ✓
```

3589202.5 / 3721 = 964.58 + 0.32 / 3721
358920.25 / 372.1 = 964.58 + 0.032 / 372.1

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Alternate Methods

The Ancient Egyptians, along with other ancient peoples, used a method of finding products, quotients, and remainders, that did not use multiplication tables. The method only uses addition, subtraction, and scaling by powers of 10 (just move the decimal point). Examples:

358920.25 / 372.1 = ?

Binary Multipliers →	Binary Products →			
		964.580	← ⊕	
1 x =	372.1	358920.250	= remaining to divide	
+	372.1	(–) 297680.000	= 372.1 x	800
2 x =	744.2	61240.250	= remaining to divide	
+	744.2	(–) 37210.000	= 372.1 x	100
4 x =	1488.4	24030.250	= remaining to divide	
+	1488.4	(–) 14884.000	= 372.1 x	40
8 x =	2976.8	9146.250	= remaining to divide	
		(–) 7442.000	= 372.1 x	20
		1704.250	= remaining to divide	
		(–) 1488.400	= 372.1 x	4
		215.850	= remaining to divide	
		(–) 148.840	= 372.1 x	0.4
		67.010	= remaining to divide	
		(–) 37.210	= 372.1 x	0.1
		29.800	= remaining to divide	
		(–) 29.768	= 372.1 x	0.08
		0.032	= remaining to divide	

358920.25 / 372.1 = 964.58 + 0.032 / 372.1

358920.25 =? 372.1 x 964.58 + 0.032

1 x =	964.58		
+	964.58		
2 x =	1929.16		
+	1929.16		
4 x =	3858.32		
+	3858.32		
8 x =	7716.64		
	964.58		
	(x) 372.1	← ⊕	
	96.458	= 964.58 x	0.1
(+)	1929.160	= 964.58 x	2
(+)	9645.800	= 964.58 x	10
(+)	19291.600	= 964.58 x	20
(+)	38583.200	= 964.58 x	40
(+)	96458.000	= 964.58 x	100
(+)	192916.000	= 964.58 x	200
	358920.218		
	(+) 0.032		
	358920.250	✓	

Choose the largest Binary Product x 10^*n* that is less than or equal to the remaining to divide, then the multiplier is the associated Binary Multiplier x 10^*n*. No guessing or trial and error required.

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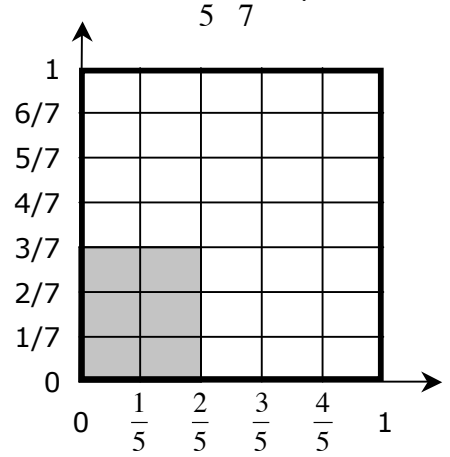
Multiplying Fractions (Area Model)

$\frac{2}{5}\cdot\frac{3}{7}=\ ?$



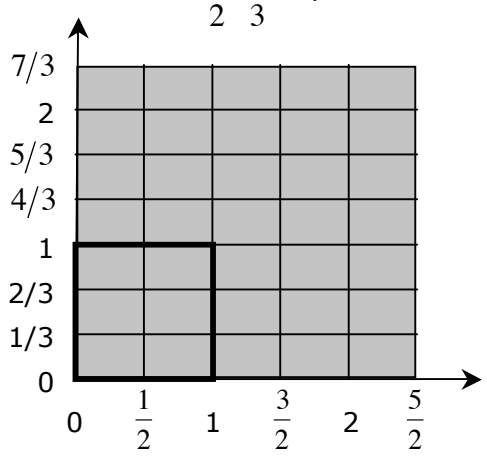
$5\cdot 7 = 35$ little boxes in unit square

$2\cdot 3 = 6$ shaded little boxes

$\Rightarrow$ shaded area is $\frac{6}{35}$ of area of unit square

$\Rightarrow \frac{2}{5}\cdot\frac{3}{7}=\frac{6}{35}=\frac{2\cdot 3}{5\cdot 7}$

$\frac{5}{2}\cdot\frac{7}{3}=\ ?$



$2\cdot 3 = 6$ little boxes in unit square

$5\cdot 7 = 35$ shaded little boxes

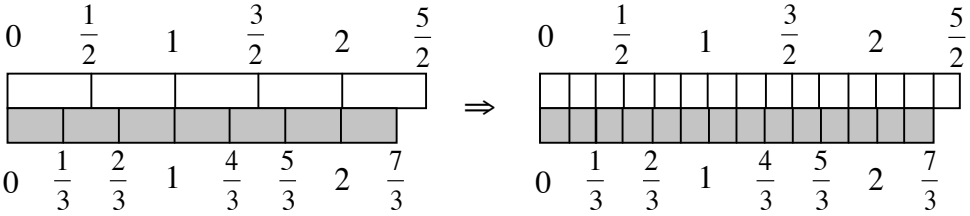
$\Rightarrow$ shaded area is $\frac{35}{6}$ of area of unit square

$\Rightarrow \frac{5}{2}\cdot\frac{7}{3}=\frac{35}{6}=\frac{5\cdot 7}{2\cdot 3}$

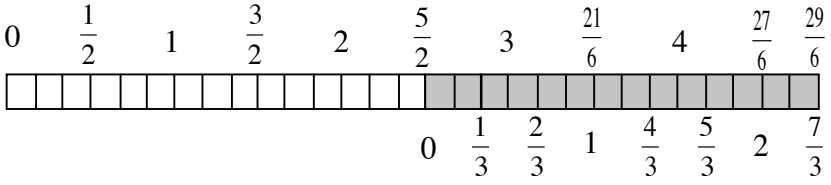
$$\Rightarrow \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}=\frac{ac}{bd}$$

Adding Fractions (Line Model, or Bread Loaf Model)

$\frac{5}{2}+\frac{7}{3}=\ ?$



$\Rightarrow$



$$\Rightarrow \frac{5}{2}+\frac{7}{3}=\frac{3}{3}\cdot\frac{5}{2}+\frac{7}{3}\cdot\frac{2}{2}=\frac{3\cdot 5+7\cdot 2}{3\cdot 2}=\frac{15+14}{6}=\frac{29}{6}$$

$$\Rightarrow \frac{a}{b}+\frac{c}{d}=\frac{d}{d}\cdot\frac{a}{b}+\frac{c}{d}\cdot\frac{b}{b}=\frac{ad+bc}{bd}$$

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