

Addition Table

10	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
9	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
8	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
7	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
6	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
5	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
4	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13
2	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12
1	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11
0	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
-1	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
-2	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
-3	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
-4	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
-5	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5
-6	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4
-7	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3
-8	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2
-9	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
-10	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
+	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

The Essentials

Identity Property of Addition: $a + 0 = a$. So zero (0) is called the Additive Identity Element.

Inverse Property of Addition: $a + (-a) = 0$. a and $-a$ are called opposites of each other.

Subtraction is the addition of the opposite: $a - b = a + (-b) = d$. Check: $a = b + d$.

	$148 + 359 + 67 = 100 + 40 + 8 + 300 + 50 + 9 + 60 + 7$	10^n place value
	$= 100 + 300 + 40 + 50 + 60 + 8 + 9 + 7$	commutative prop. (+)
12 <-- carries	$= (1 + 3) \times 100 + (4 + 5 + 6) \times 10 + (8 + 9 + 7) \times 1$	distributive prop.
148	$= (1 + 3) \times 100 + (4 + 5 + 6) \times 10 + (17 + 7) \times 1$	addition
359	$= (1 + 3) \times 100 + (4 + 5 + 6) \times 10 + (10 + 7 + 7) \times 1$	10^n place value
<u>67</u>	$= (1 + 3) \times 100 + (4 + 5 + 6) \times 10 + (1) \times 10 + (7 + 7) \times 1$	distributive prop.
574	$= (1 + 3) \times 100 + (4 + 5 + 6) \times 10 + (1) \times 10 + (14) \times 1$	addition
	$= (1 + 3) \times 100 + (4 + 5 + 6) \times 10 + (1 + 1) \times 10 + (4) \times 1$	distributive prop.
4T <-- borrow	$= (1 + 3) \times 100 + (4 + 5 + 6 + 2) \times 10 + (4) \times 1$	carry
6T <-- borrow	$= (1 + 3) \times 100 + (17) \times 10 + (4) \times 1$	addition
574	$= (1 + 3) \times 100 + (10 + 7) \times 10 + (4) \times 1$	10^n place value
(-) 289	$= (1 + 3) \times 100 + (1) \times 100 + (7) \times 10 + (4) \times 1$	distributive prop.
285	$= (1 + 3 + 1) \times 100 + (7) \times 10 + (4) \times 1$	carry
	$= (5) \times 100 + (7) \times 10 + (4) \times 1$	addition
T := 10	$= 574$	10^n place value

Multiplication Table

10	-100	-90	-80	-70	-60	-50	-40	-30	-20	-10	0	10	20	30	40	50	60	70	80	90	100
9	-90	-81	-72	-63	-54	-45	-36	-27	-18	-9	0	9	18	27	36	45	54	63	72	81	90
8	-80	-72	-64	-56	-48	-40	-32	-24	-16	-8	0	8	16	24	32	40	48	56	64	72	80
7	-70	-63	-56	-49	-42	-35	-28	-21	-14	-7	0	7	14	21	28	35	42	49	56	63	70
6	-60	-54	-48	-42	-36	-30	-24	-18	-12	-6	0	6	12	18	24	30	36	42	48	54	60
5	-50	-45	-40	-35	-30	-25	-20	-15	-10	-5	0	5	10	15	20	25	30	35	40	45	50
4	-40	-36	-32	-28	-24	-20	-16	-12	-8	-4	0	4	8	12	16	20	24	28	32	36	40
3	-30	-27	-24	-21	-18	-15	-12	-9	-6	-3	0	3	6	9	12	15	18	21	24	27	30
2	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20
1	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
-2	20	18	16	14	12	10	8	6	4	2	0	-2	-4	-6	-8	-10	-12	-14	-16	-18	-20
-3	30	27	24	21	18	15	12	9	6	3	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	-30
-4	40	36	32	28	24	20	16	12	8	4	0	-4	-8	-12	-16	-20	-24	-28	-32	-36	-40
-5	50	45	40	35	30	25	20	15	10	5	0	-5	-10	-15	-20	-25	-30	-35	-40	-45	-50
-6	60	54	48	42	36	30	24	18	12	6	0	-6	-12	-18	-24	-30	-36	-42	-48	-54	-60
-7	70	63	56	49	42	35	28	21	14	7	0	-7	-14	-21	-28	-35	-42	-49	-56	-63	-70
-8	80	72	64	56	48	40	32	24	16	8	0	-8	-16	-24	-32	-40	-48	-56	-64	-72	-80
-9	90	81	72	63	54	45	36	27	18	9	0	-9	-18	-27	-36	-45	-54	-63	-72	-81	-90
-10	100	90	80	70	60	50	40	30	20	10	0	-10	-20	-30	-40	-50	-60	-70	-80	-90	-100
X	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10

The Essentials

Identity Property of Multiplication: $n \times 1 = n$. So one (1) is called the Multiplicative Identity Element.
 Inverse Property of Multiplication: $n \times (1/n) = 1$. $1/n$ is called the Multiplicative Inverse of n .
 Division is multiplication by the inverse: $n / d = n \times (1/d) = q + r / d$. Check: $n = dq + r$.

$$358920.\underline{25} / 372.\underline{1} = 3589202.5 / 3721$$

$$358920.25 =? 372.1 \times 964.58 + 0.032$$

3721	3589202.50 = remaining to divide
(-) 3348900.00	= 3721 x 900
240302.50	= remaining to divide
(-) 223260.00	= 3721 x 60
17042.50	= remaining to divide
(-) 14884.00	= 3721 x 4
2158.50	= remaining to divide
(-) 1860.50	= 3721 x 0.5
298.00	= remaining to divide
(-) 297.68	= 3721 x 0.08
0.32	= remaining to divide

111 2	<-- x 300 carries
434 5	<-- x 70 carries
1 1 1	<-- x 2 carries
964.58	
(x) 372.1	← ⊕
96.458	= 964.58 x 0.1
(+) 1929.160	= 964.58 x 2
(+) 67520.600	= 964.58 x 70
(+) 289374.000	= 964.58 x 300
358920.218	
(+) 0.032	
358920.250	✓

Each multiplier is chosen by trial and error to produce the largest product less than or equal to the remaining to divide.

$$3589202.5 / 3721 = 964.58 + 0.32 / 3721$$

$$358920.25 / 372.1 = 964.58 + 0.032 / 372.1$$

Alternate Methods

The Ancient Egyptians, along with other ancient peoples, used a method of finding products, quotients, and remainders, that did not use multiplication tables. The method only uses addition, subtraction, and scaling by powers of 10 (just move the decimal point). Examples:

$$358920.25 / 372.1 = ?$$

$$358920.25 =? 372.1 \times 964.58 + 0.032$$

	<u>964.580</u>	←	⊕	
1 x = 372.1	358920.250	=	remaining to divide	
+ <u>372.1</u>	(-) 297680.000	=	372.1 x 800	
2 x = 744.2	61240.250	=	remaining to divide	
+ <u>744.2</u>	(-) 37210.000	=	372.1 x 100	
4 x = 1488.4	24030.250	=	remaining to divide	
+ <u>1488.4</u>	(-) 14884.000	=	372.1 x 40	
8 x = 2976.8	9146.250	=	remaining to divide	
	(-) 7442.000	=	372.1 x 20	
	1704.250	=	remaining to divide	
	(-) 1488.400	=	372.1 x 4	
	215.850	=	remaining to divide	
	(-) 148.840	=	372.1 x 0.4	
	67.010	=	remaining to divide	
	(-) 37.210	=	372.1 x 0.1	
	29.800	=	remaining to divide	
	(-) 29.768	=	372.1 x 0.08	
	0.032	=	remaining to divide	

Binary Multipliers ↑

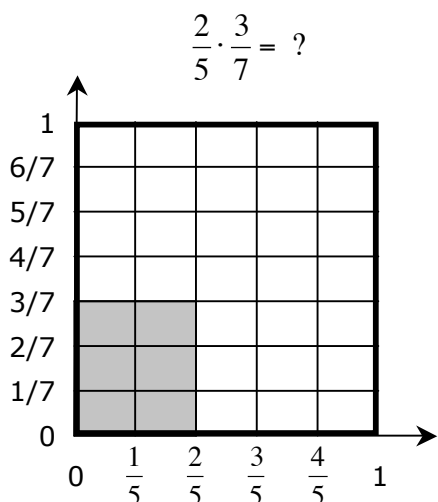
Binary Products ↑

1 x = 964.58			
+ <u>964.58</u>			
2 x = 1929.16			
+ <u>1929.16</u>			
4 x = 3858.32			
+ <u>3858.32</u>			
8 x = 7716.64			
	964.58		
	(x) <u>372.1</u>	←	⊕
	96.458	=	964.58 x 0.1
(+) 1929.160	=	964.58 x 2	
(+) 9645.800	=	964.58 x 10	
(+) 19291.600	=	964.58 x 20	
(+) 38583.200	=	964.58 x 40	
(+) 96458.000	=	964.58 x 100	
(+) <u>192916.000</u>	=	964.58 x 200	
	358920.218		
	(+) <u> 0.032</u>		
	358920.250	✓	

$$358920.25 / 372.1 = 964.58 + 0.032 / 372.1$$

Choose the largest Binary Product $\times 10^n$ that is less than or equal to the remaining to divide, then the multiplier is the associated Binary Multiplier $\times 10^n$. No guessing or trial and error required.

Multiplying Fractions (Area Model)

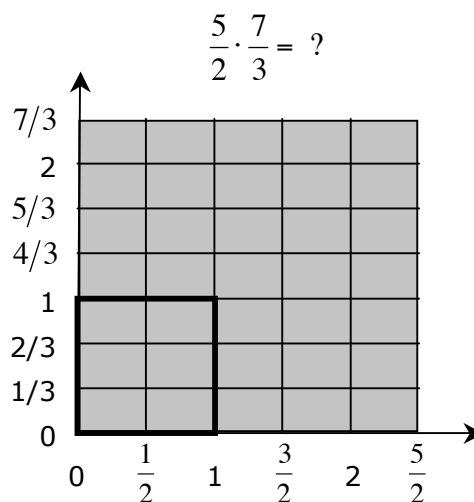


$5 \cdot 7 = 35$ little boxes in unit square

$2 \cdot 3 = 6$ shaded little boxes

\Rightarrow shaded area is $\frac{6}{35}$ of area of unit square

$$\Rightarrow \frac{2}{5} \cdot \frac{3}{7} = \frac{6}{35} = \frac{2 \cdot 3}{5 \cdot 7}$$



$2 \cdot 3 = 6$ little boxes in unit square

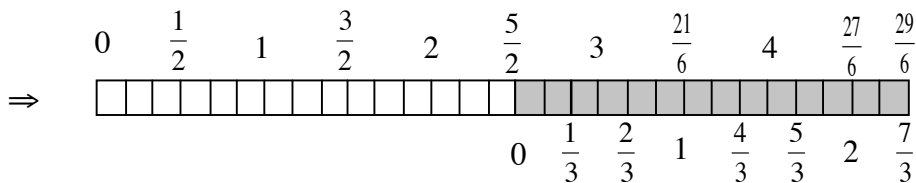
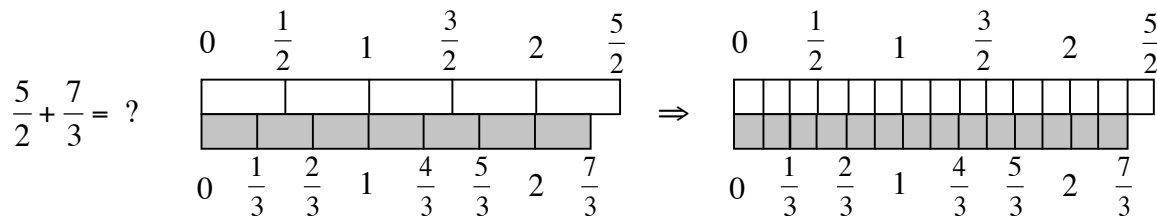
$5 \cdot 7 = 35$ shaded little boxes

\Rightarrow shaded area is $\frac{35}{6}$ of area of unit square

$$\Rightarrow \frac{5}{2} \cdot \frac{7}{3} = \frac{35}{6} = \frac{5 \cdot 7}{2 \cdot 3}$$

$$\Rightarrow \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} = \frac{ac}{bd}$$

Adding Fractions (Line Model, or Bread Loaf Model)



$$\Rightarrow \frac{5}{2} + \frac{7}{3} = \frac{3}{3} \cdot \frac{5}{2} + \frac{7}{3} \cdot \frac{2}{2} = \frac{3 \cdot 5 + 7 \cdot 2}{3 \cdot 2} = \frac{15 + 14}{6} = \frac{29}{6}$$

$$\Rightarrow \frac{a}{b} + \frac{c}{d} = \frac{d}{d} \cdot \frac{a}{b} + \frac{c}{d} \cdot \frac{b}{b} = \frac{ad + bc}{bd}$$
