Synthetic Division

I want to thank Edwin McCravy, a frequent contributor to the Algebra.com Math Board, for this extremely clear explanation not only of how to do Synthetic Division, but why it works. Synthetic is a shortcut version of long division when a polynomial is to divided by a binomial whose coefficient of x is 1.

For a tutorial explaining division of polynomials, see http://sosmath.com/algebra/factor/fac01/fac01.html

Take the long division problem:

```
         2x² -3x  +6
   -----------------
x+4)2x³ +5x² -6x +31
    2x³ +8x²
    --------
        -3x² -6x
        -3x²-12x
         -------
              6x +31
              6x +24
              ------
                   7
```

Now erase all the x's, and we have:

```
      2 - 3 + 6
  -------------
+4)2 +5 - 6 +31
   2 +8
  -----
     -3 - 6
     -3 -12
    -------
          6 +31
          6 +24
          -----
              7
```

Now erase the lower duplicates wherever a number is subtracted from itself

```
      2 - 3 + 6
  -------------
+4)2 +5 - 6 +31
     +8
    ---
     -3 - 6
        -12
     ------
          6 +31
            +24
          -----
              7
```

Now erase all the "bring-downs".

```
      2 - 3 + 6
  -------------
+4)2 +5 - 6 +31
     +8
    ---
     -3
        -12
       ----
          6
            +24
           ----
              7
```

Now this leaves a lot of room to close up the algorithm:

```
      2 - 3 + 6
  --------------
+4)2 +5  -6 +31
     +8 -12 +24
    -----------
     -3   6   7
```

Notice that the bottom row contains the -3 and the 6, the right hand numbers in the quotient, so we can erase those at the top, except for the introductory 2

```
      2
  --------------
+4)2 +5  -6 +31
     +8 -12 +24
   ------------
```

```
     -3    6    7
```

Now instead of having that introductory 2 stuck up on on top, we move it to the bottom line, under the 2, left of the -3

```
  --------------
+4)2 +5   -6 +31
     +8  -12 +24
  -------------
   2 -3    6    7
```

Let's go thru the resulting procedure. The procedure is now as follows:

1. Start with the diagram

```
  -------------
+4)2 +5   -6 +31

   ------------
```

2. Bring down the 2

3. Multiply the 2 by +4, getting +8. Write this above and to the right of 2

4. Subtract +8 from +5, getting -3. Write this below the line under the +8. Thus far we have:

```
  -------------
+4)2 +5   -6 +31
     +8
   ------------
   2 -3
```

5. Multiply the -3 by +4, getting -12. Write this above and to the right of -3

6. Subtract -12 from -6, getting +6. Write this below the line under the -12. Thus far we have:

```
  -------------
+4)2 +5   -6 +31
     +8  -12
   ------------
   2 -3   +6
```

7. Multiply the +6 by +4, getting +24. Write this above and to the right of +6, under the +31

8. Subtract +24 from +31, getting +7. Write this below the line under the +24.

```
  -------------
+4)2 +5 - 6 +31
     +8 -12 +24
   ------------
   2 -3  +6  +7
```

Now since the original polynomial had leading coefficient that of x^3, the interpretation of the bottom line, all except the last number +7, represents the quotient polynomial

$2x^2-3x+6$

and the last number +7, is the remainder.

But that's not all! There is one way to make it even easier. It is easier to add than to subtract, because subtracting involves changing signs of the second number and then adding.

Therefore we can eliminate the sign changing process and just add each time instead of subtract -- provided we will change the sign of the +4 in the beginning. So instead of

```
  -------------
+4)2 +5 - 6 +31
     +8 -12 +24
   ------------
   2 -3  +6  +7
```

we can change the sign of +4 to -4 at the beginning and then add each time instead of subtract:

```
  -------------
-4)2 +5 - 6 +31
     -8 +12 -24
   ------------
   2 -3  +6  +7
```

The final answer, then is:

$(2x^3+5x^2-6x+31)/(x+4) =$

$2x^2-3x+6 + 7/(x+4)$

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