Running Head: Ancient Scientific Calculators

Ancient Scientific Calculators

by

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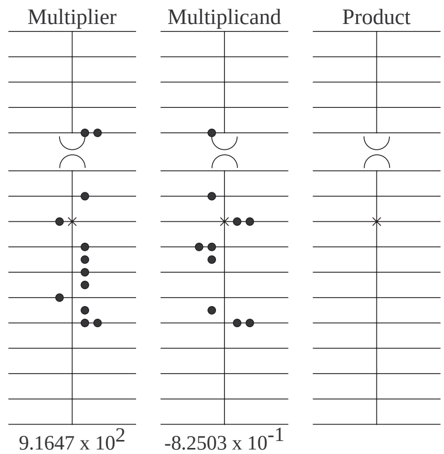
Abstract

Have you ever wondered how the Ancient Greeks, and peoples before them, did their numerical calculations? Especially their mathematicians, astronomers, scientists, engineers, and architects? Without paper and pen? Before Hindu/Arabic numerals? Whatever the methods, wouldn't they have been extremely laborious and discouraging? … *Not if the Ancients used counting boards like The Salamis Tablet, using operations described in this article!* Then the Ancients could have routinely performed all the arithmetic operations on numbers of the form $\pm a \times 10^{\pm b}$, where $1 \leq a \leq 199.99999999$ and $0 \leq b \leq 19{,}999$.

Multiplication

Figure 1



Let's start by calculating the product 916.47×-0.82503. We will use three Salamis Tablet[1] style counting boards drawn on paper and arranged side by side (Figure 1). Drawing the boards on paper allows us to slide the middle board up and down. If the boards were immobile, you would have a little more administrative work to do to match the proper lines at each stage of the calculation.

The black dots represent pebbles, the tokens used on original counting boards[2]. A pebble on a horizontal line is worth 10 pebbles on the line below. A pebble in the space between horizontal lines is worth $1/2$ of a pebble on the line above. Pebbles to the right of the vertical line are positive, to the left negative. The "×" marks the units line. The upper part of each board is for the exponent of 10.

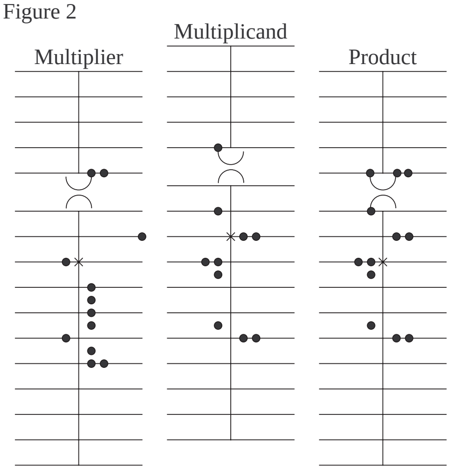
To conserve both board space and the number of pebbles needed, each digit, *n*, is represented as in this table:

n	Representation	Pebbles Needed	Unit Line Roman Numeral
0	0	0	
1	1	1	I
2	2	2	II
3	5 – 2	3	IIV[3]
4	5 – 1	2	IV
5	5	1	V
6	5 + 1	2	VI
7	5 + 2	3	VII
8	10 – 2	3	IIX[4]
9	10 – 1	2	IX

n	Representation	Pebbles Needed	Unit Line Roman Numeral
	Sum for all digits:	19	
	Average:	1.9	

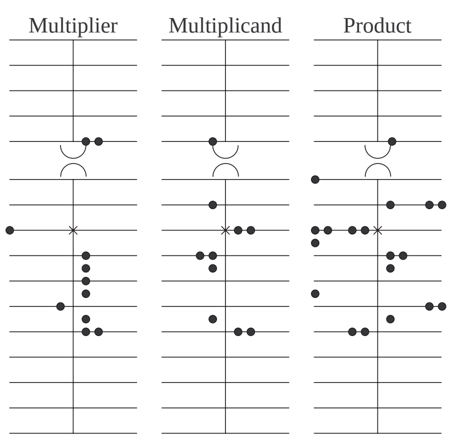
By using five's and ten's complements, there are no more than 2 pebbles on any one horizontal line or space. Enough room is left to place another number on a board without having to combine (add) the two numbers at the same time. This is a very important checkpoint that significantly reduces operator errors.

Figure 2



Let's start multiplying (Fig.2). First add the exponents. Then pick one pebble on a line of the multiplier and move it away from the median (the vertical line) to identify it. Slide the multiplicand table until its units line is collinear with the line of the identified pebble. Copy the pebbles of the multiplicand to the product table; on the same side of the median if the identified pebble is positive, on the opposite side of the median if the identified pebble is negative.

Figure 3



Slide product pebbles away from the median to make room for the next number (Fig.3). Remove identified pebble in multiplier, and identify another. Slide multiplicand board so unit's line is collinear with multiplier's identified pebble, and copy multiplicand to product board; positive if identified multiplier pebble is positive, negative otherwise.

Replace combinations of pebbles on any line of product with most efficient representation (Fig.4). Slide pebbles away from the median to make room to add next partial product. Remove identified pebble in multiplier, identify another. Repeat process until no more pebbles exist on multiplier lines (Fig.5-8).

Figure 4

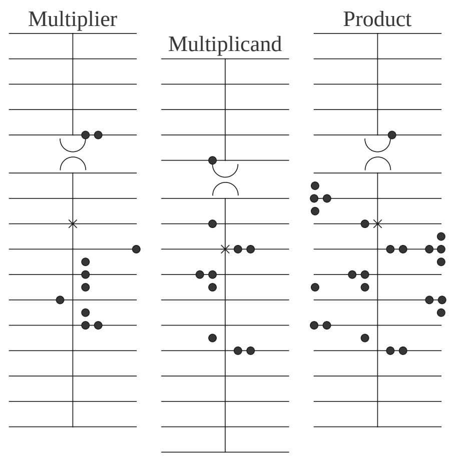


Figure 5

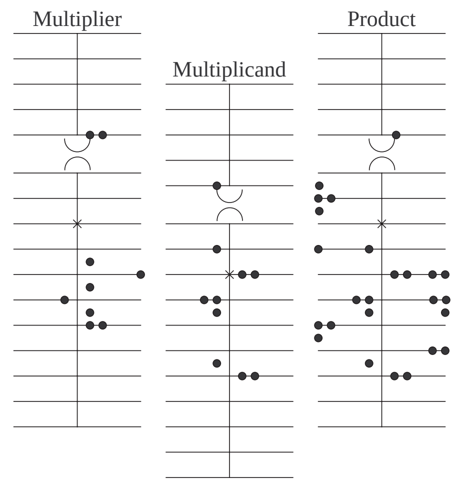


Figure 6

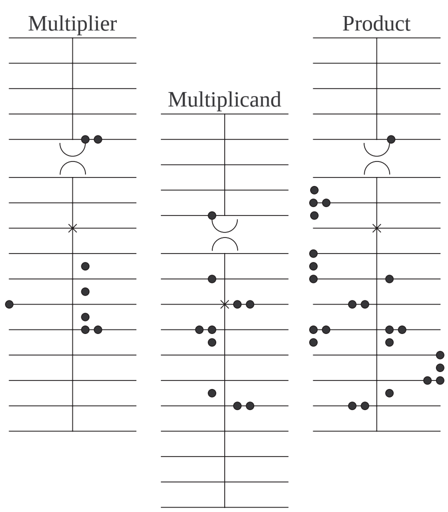
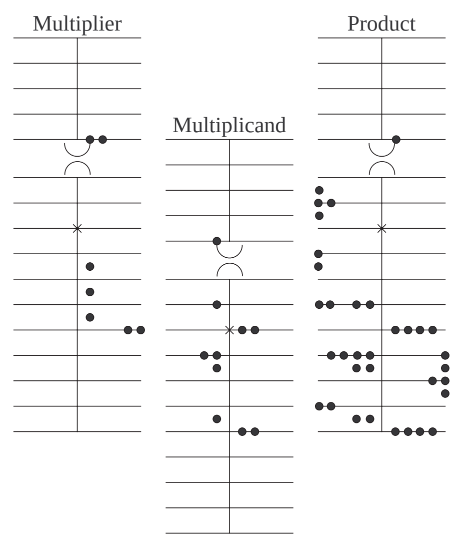


Figure 7



Now let's double the Multiplier and halve the Multiplicand (Fig.9). This is easily accomplished on a pebble by pebble basis: double by duplicating every pebble, halve by changing 1 to 1/2 and 5 to 2 + 1/2; then reduce to minimum representations. Do the halving by starting at the bottom line and working up; same for reducing to minimum representations.

Figure 8

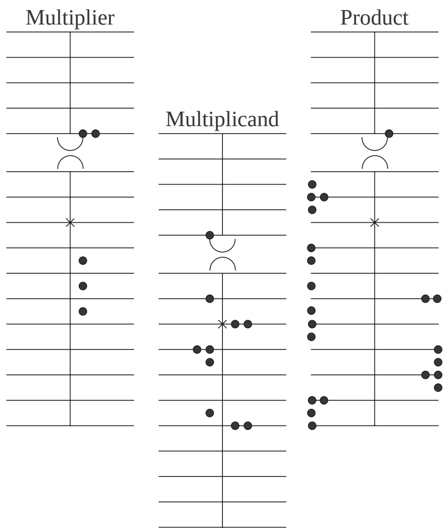


Figure 9

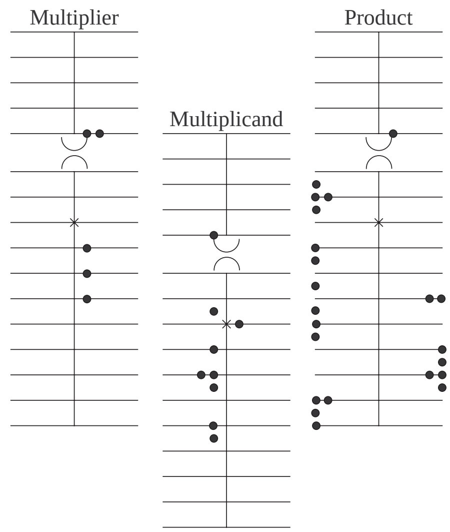


Figure 10

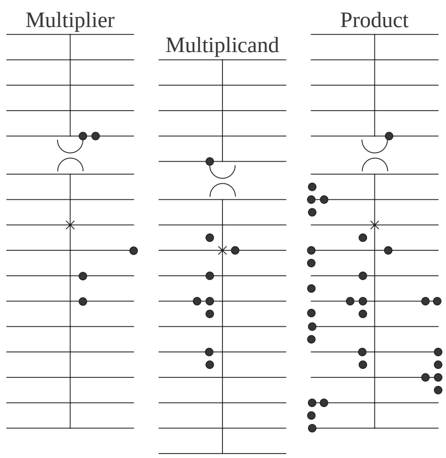


Figure 11

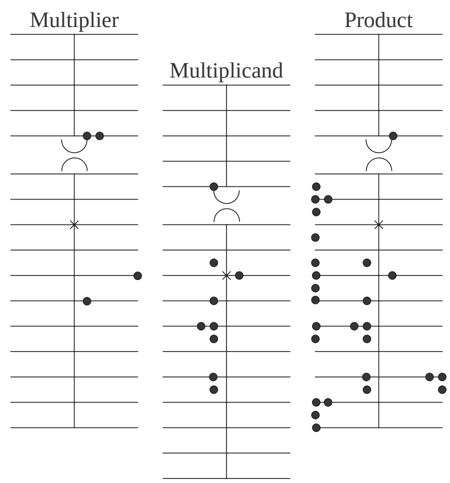


Figure 12

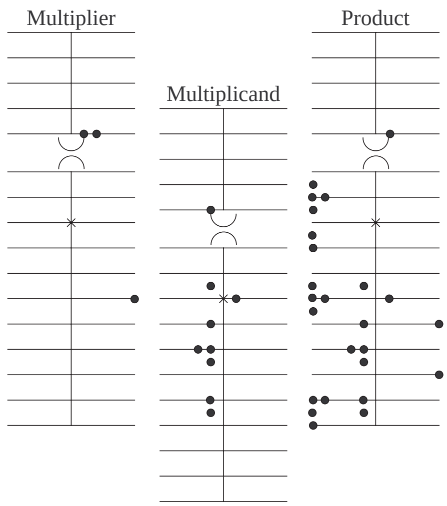


Figure 13

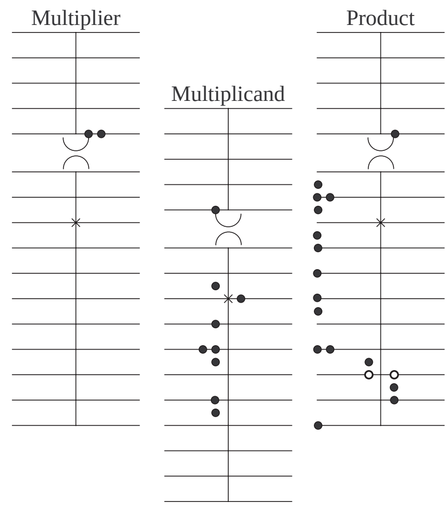
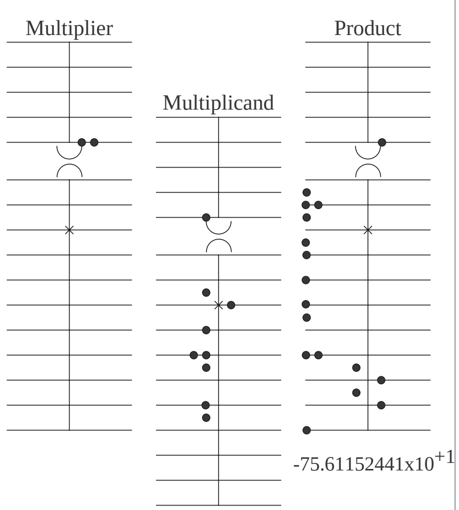


Figure 14



In Figure 13 the white pebbles are added to facilitate forming readable digits. Reading the Product board in Figure 14 from the top down, the answer is

$$-75.61152441 \times 10^{+1} = -756.1152441.$$

Without using multiplication or addition tables, memorized or otherwise, we calculated a product of two five-digit numbers, yielding ten digits of precision, using only 50 pebbles and lines drawn in the sand! It's like writing an essay with paper and pen instead of computer, word processor, and printer. A little more time consuming, but the equipment is much cheaper.

Division

Division is a very similar process, except you keep a count of how many decimal fractions of the divisor you can subtract from the dividend until it is exhausted. The count is the quotient.

Pebble Count Efficiency

A fully populated counting board would have two pebbles on every line and one on every space, for a total of 32 + 14 = 46 pebbles. We need four boards worth of pebbles to do multiplication or division, so we should be able to do any problem with no more than 46 x 4 = 184 pebbles. But if you assume that digits in a random number occur randomly, then it would take (10+4) × 1.9 × 4 = 106.4 pebbles. My "pebbles" are pennies and my bag contains $1.10 worth.

If you do not allow negative parts, the digit representation table becomes:

n	Representation	Pebbles Needed	Unit Line Roman Numeral
0	0	0	
1	1	1	I
2	2	2	II
3	3	3	III
4	4	4	IIII
5	5	1	V
6	5 + 1	2	VI
7	5 + 2	3	VII
8	5 + 3	4	VIII
9	5 + 4	5	VIIII
	Sum for all digits:	25	
	Average:	2.5	

Here's an interesting investigation for students: In both representations, how high can you count sequentially on a counting board if you only have *k* pebbles? What patterns do the answers contain?

Here's what happens:

	without neg. parts		with neg. parts	
k	Maximum Sequential Count	Increment	Maximum Sequential Count	Increment
0	0	0	0	0
1	1	1	1	1
2	2	1	2	1
3	3	2	12	10
4	8	5	22	10
5	18	10	72	50
6	28	10	172	100
7	38	10	272	100
8	48	10	772	500
9	98	50	1,772	1,000
10	198	100	2,772	1,000
11	298	100	7,772	5,000
12	398	100	17,772	10,000
13	498	100	27,772	10,000
14	998	500	77,772	50,000
15	1,998	1,000	177,772	100,000
16	2,998	1,000	277,772	100,000
17	3,998	1,000	777,772	500,000
18	4,998	1,000	1,777,772	1,000,000
19	9,998	5,000	2,777,772	1,000,000
20	19,998	10,000	7,777,772	5,000,000
…				
31	2,999,998	1,000,000	27,777,777,772	10,000,000,000

So the representation using negative parts is much more efficient, adding 10 billion to the maximum count from 30 to 31 pebbles, vs. 1 million for the representation without negative parts. We stop at 31 pebbles because the number 277.77777772×10^8 fills the large table on The Salamis Tablet. The number's configuration is interesting: a 2 followed by two 7's for the integer part, and seven 7's followed by a 2 for the fractional part. Wouldn't these results elevate the number 7 to a very special, even mystical, place in the minds of the ancients?

Archimedes

Archimedes was born in 287 BC and died in 212 BC. The Salamis Tablet has been dated to the third or fourth century BC. Could Archimedes have been involved in The Salamis Tablet's creation? (Not the counting board, just the actual marble Salamis Tablet.)

> *The Sand Reckoner* is probably the most accessible work of Archimedes … In this work, Archimedes sets himself the challenge of debunking the then commonly held belief that the number of grains of sand is too large to count. In order to do this, he first has to invent a system of naming large numbers in order to give an upper bound, and he does this by starting with the largest number around at the time, the myriad myriad or one hundred million **(a myriad is 10,000).** Archimedes' system goes up to 10^{8* 10^{16}} which is a myriad myriad to the myriad myriadth power, all taken to the myriad myriadth power. (Retrieved 6/26/05 from http://en.wikipedia.org/wiki/The_Sand_Reckoner)

The Salamis Tablet's 11 line large table can accommodate multiples of 100 myriad-myriad, and its 5 line small table can accommodate multiples of 1 myriad. (The 31-pebble efficiency using negative parts, above, is a myriad times more efficient than not using negative parts.)

The Salamis Tablet is a Monument, Not a Working Counting Board

The Salamis Tablet is a marble slab 4.9 feet long, 2.5 feet wide, and 1.75 inches thick. That's a monument, not a working device. No others have been found. So if a monument, why a monument? Because it documents huge advances in the ability of ancient peoples to do arithmetic calculations; serious and necessary calculations in mathematics, astronomy, science, engineering, architecture, empire sized government accounting and taxation systems, and commercial accounting.

Earlier Counting Boards

But when did these advances in calculation methods appear? How much earlier than the creation of The Salamis Tablet? The Egyptians had a decimal number system from 2700 BC, and the Babylonians had a place value sexagesimal number system from 2000 BC. Did their counting boards have the features of The Salamis Tablet?

The Salamis Tablet can be easily modified to do sexagesimal arithmetic while still using the pebble saving technique of using positive and negative parts for appropriate digits, just replace every second line from the bottom with a dashed line. Then a pebble on a solid line is worth 60 pebbles on the solid line below it. A pebble on a dashed line is worth 10 pebbles on the solid line below it. A pebble in a space is worth 1/2 a pebble on either kind of line immediately above it.

The resulting Sexagesimal Counting Board accommodates 5 digits in the large table and 2 digits in the small table. These numbers of digits make a lot of sense! One units digit, four fractional digits (one thumb and four fingers), and 2 digits for an exponent (one digit would be too few). The Babylonians are credited with the invention of the first counting boards; was this Sexagesimal Counting Board the one they used? If so, The Salamis Tablet would be a direct descendent of such a board; **in fact, an exact copy!** References to the Salamis Tablet often say that it was used by the Babylonians, even though there are Greek inscriptions on it. Why? Are there sources documenting this? If true, the Greeks copied the technology; and Archimedes probably got the idea of exponents from the Babylonians. Two questions for historians: how often did Babylonians write sexagesimal numbers with more than 5 significant digits; and, how necessary would a zero symbol for the ends of numbers be if all numbers were registered on a sexagesimal counting board in what we now call scientific notation?

Clues That Led to These Conjectures

The semicircles on the top of the large table and at the bottom of the small table of The Salamis Tablet are halves of a whole circle, the "perfect" geometric shape. So the two halves must indicate that their tables are part of the same whole, the same number. Likewise, the ends of each semicircle are pointing to the left and right sides of their table, again indicating that the two halves are connected; are one.

Many philosophies teach that "things" occur in pairs of opposites: male / female, positive / negative, Yin / Yang, … Wouldn't opposite numbers make sense, then, to the Ancients?

A manual for the Japanese Soroban teaches the use of 5 and 10 complements in forming and working with numbers on the abacus. If for abacii, why not for counting boards?

The Soroban and the Roman Hand Abacus both have a 5-count bead above 4 one-count beads. This leads naturally to using the space between lines on a counting board for pebbles worth 1 / 2 those on the line above.

Roman Numerals are naturals for setting up a counting board or for recording the results. They certainly can't be used easily for calculations. So the Romans must have used counting boards in the manner described here.

Egyptian numeral hieroglyphs are also naturals for setting up a Salamis Tablet counting board or for recording the results, if you assume Egyptians recorded their numbers only as positive digits.

Babylonian cuneiform numbers are naturals for setting up a sexagesimal counting board. Like the Romans, they also used subtractive notation.

References

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Endnotes

[1] See http://www.ee.ryerson.ca/~elf/abacus/history.html#salamis. Retrieved June 27, 2005.

[2] "The Roman expression for 'to calculate' is 'calculus ponere' - literally, 'to place pebbles'. When a Roman wished to settle accounts with someone, he would use the expression 'vocare aliquem ad calculos' - 'to call them to the pebbles.'" Retrieved July 3, 2004 from http://mathforum.org/library/drmath/view/57572.html.

[3] Three would be entered on a counting board as IIV, but would be written as III since that form is simpler. I wonder if there are any written examples of IIV (which would be scribe errors).

[4] "… constructions such as IIX for eight have been discovered." Retrieved July 2, 2004, from http://en.wikipedia.org/wiki/Roman_numeral. Also retrieved July 2, 2004: http://www2.inetdirect.net/~charta/Roman_numerals.html contains a page from Pietro Bongo's *Mysticae Numerorum Significationis Liber* that clearly shows 8,000 = (I)(I)((I)); i.e., 8 being represented in the form IIX. Even the date, 1584 = (I) I) XXCIV, on the book's title page at http://www2.inetdirect.net/~charta/tp.html demonstrates the usage.